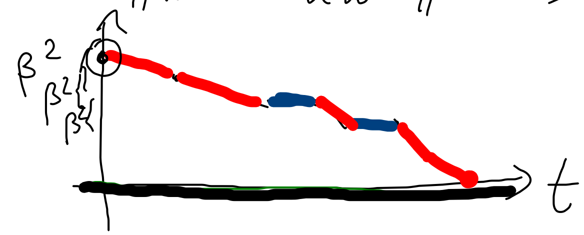
$w^{(1)} = 0$

$\rightarrow \quad w^{(t+1)} \leftarrow w^{(t)} \quad$ se $\quad y w^{(t)T} x > 0$

$w^{(t+1)} \leftarrow w^{(t)} + \underbrace{\eta}_{=1}\, y \cdot x \quad$ se $\quad y w^{(t)T} x \leq 0$

$\|w^{(t)} - \alpha w^*\|^2 \geq 0$



$w^{(1)}, w^{(2)}, \ldots, w^{(k)}, \ldots\ldots$

Sia w^* un vettore che separa perfettamente gli esempi : $y w^{*T} x > 0$ (per ogni (x,y))

Idea : monitorare la quantità $\|w^{(t)} - \alpha w^*\|^2$ al variare di t (per qualche $\alpha > 0$)

Se $y w^{(t)T} x > 0$, $\|w^{(t+1)} - \alpha w^*\|^2 = \|w^{(t)} - \alpha w^*\|^2$

Se $\boxed{y w^{(k)T} x \leq 0}$ (al passo k l'esempio non era classificato correttamente)

Per def. dell'algoritmo, in questo caso $w^{(k+1)} = w^{(k)} + y \cdot x$

$\|w^{(k+1)} - \alpha w^*\|^2 = \|w^{(k)} - \alpha w^* + y x\|^2 = \langle (w^{(k)} - \alpha w^*) + (y x), (w^{(k)} - \alpha w^*) + (y x) \rangle$

$= \langle w^{(k)} - \alpha w^*, w^{(k)} - \alpha w^* \rangle + \langle y x, y x \rangle + \langle w^{(k)} - \alpha w^*, y x \rangle + \langle y x, w^{(k)} - \alpha w^* \rangle$

$= \|w^{(k)} - \alpha w^*\|^2 + \|\cancel{y} x\|^2 + 2 \underline{\langle w^{(k)} - \alpha w^*, y x \rangle}$

$$2\langle w^{(k)} - \alpha w^*, yx\rangle = 2\overbrace{\boxed{\langle w^{(k)}, yx\rangle}}^{\leq 0} - 2\alpha\langle w^*, yx\rangle \leq -2\alpha\langle w^*, yx\rangle$$

$$\Rightarrow \|w^{(k)} - \alpha w^*\|^2 + \|\cancel{y}x\|^2 + 2\underline{\langle w^{(k)} - \alpha w^*, yx\rangle}$$

$$\leq \|w^{(k)} - \alpha w^*\|^2 + \|x\|^2 - 2\alpha\langle w^*, yx\rangle$$

Osservo che $\langle w^*, yx\rangle = y \cdot w^{*T} x > 0$ per ogni (x,y)

Definisco

$$\beta^2 = \max_{k=1}^{m} \|x^{(k)}\|^2$$

$$\gamma = \min_{k=1}^{m} y^{(k)} w^{*T} x^{(k)} > 0$$

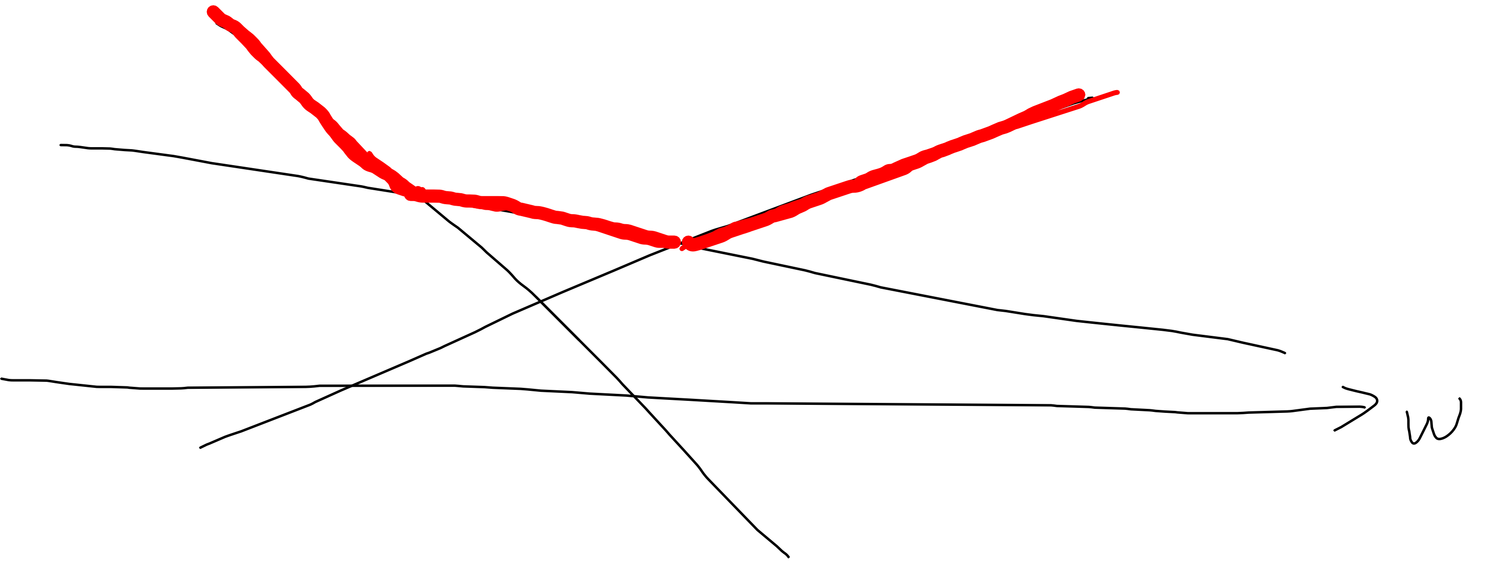
$$\alpha = \beta^2/\gamma \Rightarrow \alpha\gamma = \beta^2$$

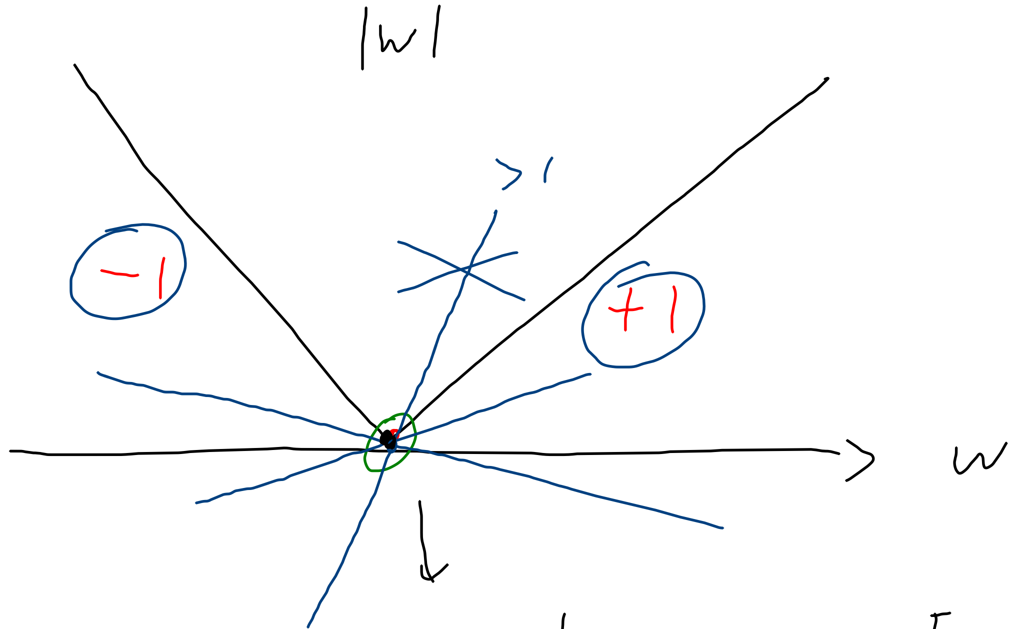
$$\|w^{(k+1)} - \alpha w^*\|^2 \leq \|w^{(k)} - \alpha w^*\|^2 + \beta^2 - 2\alpha\gamma$$

$$= \|w^{(k)} - \alpha w^*\|^2 + \beta^2 - 2\beta^2$$

$$= \|w^{(k)} - \alpha w^*\|^2 - \beta^2$$

Quindi: ogni volta che l'alg. incontra un esempio non correttamente classificato, la quantità $\|w^{(k)} - \alpha w^*\|^2$ $\boxed{\text{diminuisce di } \beta^2}$ (almeno)


w



qualunque $g \in [-1, 1]$