$$\mathrm{Cov}(x_i, x_j)$$

$$\rightarrow \mathrm{Cov}(x_j, x_i) = \mathrm{Cov}(x_i, x_j)$$

$$\mathrm{Cov}(x_i, \gamma x_j) = \gamma\, \mathrm{Cov}(x_i, x_j)$$

$$\mathrm{Cov}(x_i + z, x_j) = \mathrm{Cov}(x_i, x_j) + \mathrm{Cov}(z, x_j)$$

$$\mathrm{Cov}(x_i, x_i) = \mathrm{Var}(x_i) \geq 0$$

$$\boxed{|\mathrm{Cov}(x_i, x_j)| \leq \sqrt{\mathrm{Var}(x_i)} \cdot \sqrt{\mathrm{Var}(x_j)}}$$

$$\Longleftarrow$$

$$\langle a, b\rangle = \langle b, a\rangle$$

$$\langle a, \gamma b\rangle = \gamma \langle a, b\rangle$$

$$\langle a+b, c\rangle = \langle a, c\rangle + \langle b, c\rangle$$

$$\langle a, a\rangle \geq 0$$

ede' $= 0 \iff a = \vec{0}$

Cauchy-Schwarz:

$$|\langle a, b\rangle| \leq \underbrace{\|a\|}_{= \langle a,a\rangle^{1/2}} \cdot \underbrace{\|b\|}_{= \langle b,b\rangle^{1/2}}$$

$$\Rightarrow -1 \leq \boxed{\frac{Cov(x_i, x_j)}{\sqrt{Var(x_i)}\sqrt{Var(x_j)}}} \leq 1 \rightarrow \text{Coefficiente di correlazione tra } x_i \text{ e } x_j$$

$\Vert$

coeff. correlaz.

$$\rightarrow \hat{\mu} = \frac{1}{m} \sum_{k=1}^{m} x^{(k)}$$

$$\hat{\Sigma}_{ij} = \frac{1}{m} \sum_{k=1}^{m} (x_i^{(k)} - \hat{\mu}_i)(x_j^{(k)} - \hat{\mu}_j) \qquad \forall i,j = 1..d$$

$$\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu) =$$

$$= \frac{1}{2}x^T\Sigma^{-1}x \underbrace{- \frac{1}{2}x^T\Sigma^{-1}\mu - \frac{1}{2}\mu^T\Sigma^{-1}x}_{= -x^T\Sigma^{-1}\mu} + \frac{1}{2}\mu\Sigma^{-1}\mu^T$$