$$\arg\min_{w\in\mathbb{R}^{d+1}} \frac{1}{m}\|Xw-y\|^2 + \lambda\|\omega\|^2 = \arg\min_{w\in\mathbb{R}^{d+1}} \|Xw-y\|^2 + \lambda m\|\omega\|^2$$

$$\omega = (w_1, \dots, w_d) \qquad w = (\underbrace{w_0, \dots, w_d}_{d+1})$$

$$I_0 = \underbrace{\begin{bmatrix} 0 & & & 0 \\ & 1 & & \\ & & 1 & \\ & & & \ddots \\ 0 & & & & 1 \end{bmatrix}}_{d+1} \Big\} d+1$$

$$\|\omega\|^2 = w_1^2 + w_2^2 + \dots + w_d^2$$
$$= 0\cdot w_0^2 + 1\cdot w_1^2 + \dots + 1\cdot w_d^2$$
$$= \boxed{w^T I_0 w} \qquad \begin{pmatrix} 0 \\ w_1 \\ w_2 \\ \vdots \\ w_d \end{pmatrix} = w_1^2 + \dots + w_d^2$$

$$C \geq 0$$

$$= \arg\min_{w\in\mathbb{R}^{d+1}} \underbrace{(Xw-y)^T(Xw-y) + \lambda m\, w^T I_0 w}_{g(w)}$$

convessa (I_0 ha autovalori ≥ 0)

$$\underbrace{w^T \underbrace{X^T X}\, w}_{\text{convessa}} - \underbrace{2 w^T X^T y + y^T y}_{\text{lineare}}$$

$\Rightarrow$ $g(w)$ è convessa

Minima quando $\nabla g(w) = 0$

$$g(w) = w^T X^T X w - 2(w^T)(X^T y) + \underbrace{y^T y}_{\text{sparisce}} + C\, w^T I_0\, w$$

$$\nabla g(w) = 2 X^T X w - 2 X^T y + 0 + 2 C I_0 w$$

Quindi $\nabla g(w) = 0 \iff \cancel{2} X^T X w - \cancel{2} X^T y + \cancel{2} C I_0 w = 0_{d+1}$

$$\iff (X^T X + C I_0) \cdot w = X^T y$$

$$w = (X^T X + \underbrace{C}_{\lambda m} I_0)^{-1} X^T y$$