Disuguaglianza di Hoeffding

Siano $\vartheta_1, \vartheta_2, \dots, \vartheta_m$ variabili aleatorie indipendenti e identic. distribuite

con $\mathbb{E}[\vartheta_i] = \mu$ per $i = 1, \dots, m$

e $\Pr[a \leq \vartheta_i \leq b] = 1$ per $i = 1, \dots, m$

Allora: (per ogni $\varepsilon > 0$)

$$\Pr\left[\left|\frac{1}{m}\sum_{i=1}^{m}\vartheta_i - \mu\right| > \varepsilon\right] \leq 2\, e^{-2m\varepsilon^2/(b^2-a^2)}$$

Prendiamo $\quad \vartheta_i = \ell(h, (x^{(i)}, y^{(i)})) \quad i = 1, \dots, m$

$$\mu = \mathbb{E}[\vartheta_i] = \mathop{\mathbb{E}}_{(x,y)\sim D} \ell(h, (x, y)) = L_D(h)$$

$i = 1, \dots, m$

$a = 0 \qquad b = b \qquad \Pr[0 \le \vartheta_i \le b] = 1$ ✓

$$\varepsilon = b \sqrt{\frac{\log(2/\delta)}{2m_T}}$$

$\log(\delta/2)$

$$\Pr\left[\left| L_T(h) - \underbrace{L_D(h)}_{\mu} \right| > \varepsilon\right] \le 2 \exp\left(+ \frac{\cancel{2m_T}}{\cancel{b^2}} \cdot \cancel{b^2} \frac{\log(2/\delta)}{\cancel{2m_T}} \right)$$

$$= \delta .$$