CORRIGENDUM TO : CLASSIFICATION OF VARIETIES WITH CANONICAL CURVE SECTION VIA GAUSSIAN MAPS ON CANONICAL CURVES

[AMER. J. MATH. 120(1998), NO. 1, 1-21]

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Abstract. We correct a mistake in the statement and proof of Lemma 2.3(d) in [Amer. J. Math. 120(1998), no. 1, 1-21]. This in turn implies a change in Table 2.14.

As observed by B. Totaro [T, Example 5.2], in the proof of [CLM, Lemma 2.3(d)] there is an error.

Consequently the statement of [CLM, Lemma 2.3(d)] needs to be changed as follows:

- $S_{r,2}$ has smooth ramification divisor, $g = 2$ and r, k are such that $rk \geq 7$.

As for the proof, it is enough carry on the same proof using that $N_\pi \cong \pi^* \mathcal{O}_B(6)$ (and not $\pi^* \mathcal{O}_B(3)$ as erroneously written on page 6).

Note that the proof now shows that $H^1(S_{r,2}, \Omega^1_{S_{r,2}}(rk)) = 0$ if and only if $rk \geq 7$.

The above change in the statement of [CLM, Lemma 2.3(d)] also affects the cases $g = 2$ in [CLM, Table 2.14]. Namely, Table 2.14 needs to be changed as follows:

- remove the case $r = 4, g = 2$;
- replace the case $r \geq 5, g = 2$ with $r \geq 7, g = 2$.

Note that [CLM, Table 2.14] is used in [CLM, Thm. 2.15] and in [CLM, Prop. 2.22].

On the other hand we point out that all other results in [CLM] are unaffected by these changes.

About [CLM, Prop. 2.22], we remark that the surjectivity of the Gaussian map $\Phi_{\omega_{C_{r,2}}, \omega^{\otimes k}_{C_{r,2}}}$ (which is equivalent to $h^0(N_{C_{r,2}}(-k)) = 0$) still holds for $k \geq 2$ and $r \geq 4$. In fact when $r \geq 5$ the proof of [CLM, Prop. 2.22] works unchanged. When $r = 4$ it follows by the ensuing

Claim 0.1. *Let $\pi : S_2 \to \mathbb{P}^2$ be a double cover ramified along a smooth sextic and let $H = \pi^* \mathcal{O}_{\mathbb{P}^2}(1)$. Let C be any smooth curve in $|4H|$. Then*

(i) $\mathrm{Cliff}(C) = 4$

(ii) $\Phi_{\omega_C, \omega_C^{\otimes k}}$ *is surjective for any $k \geq 2$.*

Proof. From the exact sequence

$$0 \to -3H \to H \to H_{|C} \to 0$$

we get that $h^0(H_{|C}) = h^0(H) = 3$ so that $|H_{|C}|$ contributes to the Clifford index of C and then $\mathrm{Cliff}(C) \leq \mathrm{Cliff}(H_{|C}) = 4$. Assume that $\mathrm{Cliff}(C) \leq 3$. By [K, Lemma 8.3] there is an effective divisor D on S_2 such that $0 \leq D^2 \leq \mathrm{Cliff}(C) + 2 \leq 5$ and $\mathrm{Cliff}(C) = 4H \cdot D - D^2 - 2$. Since we are on a K3 surface, we are left with the three possibilities $D^2 = 0, 2, 4$.

If $D^2 = 0$ we first observe that $D \cdot H \geq 2$. In fact if $D \cdot H = 1$ then $H \cdot (D - H) = -1$, hence $H^0(D - H) = 0$. Let Γ be any smooth curve in $|H|$ and consider the exact sequence

$$0 \to D - H \to D \to D_{|\Gamma} \to 0.$$

We get by Riemann-Roch that $h^0(D_{|\Gamma}) \geq h^0(D) \geq 2$. But this is not possible since Γ is not rational. Therefore $D \cdot H \geq 2$ and $\mathrm{Cliff}(C) \geq 6$, a contradiction.

If $D^2 = 2$ (respectively 4), then $D \cdot H \geq 2$ (respectively 3) by the Hodge index theorem, thus giving $\mathrm{Cliff}(C) = 4H \cdot D - D^2 - 2 \geq 4$, again a contradiction. This proves (i).

Finally (ii) is a consequence of (i) and [BEL, Thm. 2] or [KL, Prop. 2.11]. □

References

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