Esercizi su integrali

1. Calcolare i seguenti integrali indefiniti:

$$\int \frac{1}{e^x + e^{-x}}\mathrm{d}x; \qquad \int \frac{1}{\sqrt[3]{x}-1}\mathrm{d}x; \qquad \int \sin^3 x\,\mathrm{d}x; \qquad \int x\cos(2x)\mathrm{d}x; \qquad \int \arcsin x\,\mathrm{d}x.$$

2. Calcolare i seguenti integrali definiti:

$$\int_1^e \frac{\ln^2 x}{x}\mathrm{d}x; \qquad \int_2^4 \frac{1}{x^3-x}\mathrm{d}x; \qquad \int_0^1 x\,\mathrm{arctg}x\,\mathrm{d}x; \qquad \int_0^{\frac{\pi}{6}} \frac{1}{1-\sin(2x)}\mathrm{d}x; \qquad \int_{-1}^1 (1+\mathrm{tg}x)e^{|x|}\mathrm{d}x.$$

3. Discutere la convergenza dei seguenti integrali impropri:

$$\int_0^1 \frac{1}{\sqrt{x}+x^\pi}\mathrm{d}x; \qquad \int_0^1 \frac{1-\cos x}{x^2}\mathrm{d}x; \qquad \int_0^1 \frac{|\ln x|}{x+1}\mathrm{d}x; \qquad \int_0^1 \frac{e^{-x}-e^{-2x}}{x-\sin x}\mathrm{d}x; \qquad \int_0^1 \frac{\mathrm{arctg}\frac{1}{x}}{\mathrm{arctg}x}\mathrm{d}x.$$

$$\int_1^{+\infty} \frac{1}{\sqrt{x}+x^\pi}\mathrm{d}x; \qquad \int_1^{+\infty} \frac{1-\cos x}{x^2}\mathrm{d}x; \qquad \int_1^{+\infty} \frac{|\ln x|}{x+1}\mathrm{d}x; \qquad \int_1^{+\infty} \frac{e^{-x}-e^{-2x}}{x-\sin x}\mathrm{d}x; \qquad \int_1^{+\infty} \frac{\mathrm{arctg}\frac{1}{x}}{\mathrm{arctg}x}\mathrm{d}x.$$