MOTIVES OF ALGEBRAIC VARIETIES AND THE CONJECTURES OF BEILINSON AND MURRE

by

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We consider the category of *Chow motives* $\mathcal{M}_{rat}(k)$ over a perfect field k with $\mathbb{Q}$ -coefficients.This is a pseudoabelian, $\mathbb{Q}$-linear, tensor category which may be embedded as a full subcategory in the triangulated category of motivic complexes $DM(k)$ constructed by Voevodsky.In any pseudoabelian category one has the notions of *finite dimensional* and *Schur finite dimensional* objects. We show how these notions are related to the Conjectures of Beilinson and Murre about the existence of a suitable filtration on the Chow groups $F^{\bullet}A^j(X)$ of a smooth projective variety X, where $A^j(X)$ is the group of codimension j cycles, modulo rational equivalence with $\mathbb{Q}$-coefficients.

In particular one has the following result :

Let X be a smooth projective surface over $\mathbb{C}$ with $p_g = 0$. then Bloch's Conjecture holds for X, i.e the Albanese Kernel vanishes iff the motive of X is finite dimensional in $\mathcal{M}_{rat}(\mathbb{C})$. This is in turn euivalent to the existence of an open subset U of X such that its motive is Schur finite dimensional in the category $DM(\mathbb{C})$

Since it s known that finite dimensionality is a birational invariant for the motive of a surface and all surfaces with $p_g = 0$ and which are not of general type have a finite dimensional motive, the above result in particular yields a purely motivic proof of Bloch's Conjecture for surfaces with $p_g = 0$ and Kodaira dimension < 2.

It also shows that Bloch's Conjecture holds for some classes of surfaces of general type with $p_g = 0$ like Godeux surfaces and surfaces X with $K_X^2 = 8$ and such that there exists an involuton β on X with X/β a rational surface.

More generally: if X is surface with $p_g = q = 0$,such that $A^2(X)$ is trivial then there exists a rational surface Y whose motive is isomorphic (up to torsion) to the motive of X.