**Quantum Cohomology of $\mathbf{Hilb}^2(\mathbb{P}^1 \times \mathbb{P}^1)$ and enumerative applications** - Dalide Pontoni

I describe the Hilbert scheme $\mathrm{Hilb}^2(\mathbb{P}^1 \times \mathbb{P}^1)$ and its Big and Small Quantum Cohomology rings, $QH^*$ and $QH^*_s$ respectively. Both of them are deformations of the usual cohomology ring $H^*(\mathrm{Hilb}^2(\mathbb{P}^1 \times \mathbb{P}^1), \mathbb{Q})$. They are obtained by introducing some formal variables and defining the so called $*$-product which extends the usual $\cup$-product. To give an explicit presentation of $QH^*_s$ I need to know almost all the Gromov-Witten Invariants of $\mathrm{Hilb}^2(\mathbb{P}^1 \times \mathbb{P}^1)$. These invariants are enumerative; they encode information on hyperelliptic curves on $\mathbb{P}^1 \times \mathbb{P}^1 \subseteq \mathbb{P}^3$ of given genus and bidegree satisfying some incident conditions. I present a result about this enumerative significance.