Presenting small Quantum Cohomology of some smooth Fano threefolds

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In the talk we describe how we computed explicit presentations for small Quantum Cohomology rings of some Fano threefolds which are obtained as one- or two-curve blow-ups from the projective space or the quadric. Some computations were carried out with the aid of specific commutative algebra software. We remark that because of systematic usage of the associativity property of quantum product, we needed to compute geometrically only a very small and enumerative set of Gromov-Witten invariants.

Explicit presentations helped in studying a conjecture of Dubrovin about generic semisimplicity of Quantum Cohomology. By examining our data together with pre-existing computations, we checked semisimplicity for 36 of the 59 smooth Fano threefolds having $b_3 = 0$; on the other hand, since the bounded derived categories of coherent sheaves of these threefolds have a complete exceptional set of a certain length, the conjecture holds for them.