Thursday, February 17, 2022 2:05 PM

Aim

- Let $\overline{\mathcal{M}}_{g,n}$ be the moduli stack of stable curves of genus g and n markings.
  $\overline{\mathcal{M}}_{g,n}$ is a smooth, irreducible, proper DM (= Deligne Mumford) stack of dim $3g-3+n$ which admits a projective coarse moduli space.

- C smooth proj. curve and let $\mathrm{Bun}^{ss}_{r,d}(C)$ be the moduli stack of semistable vector bundles of rank r and degree d on C. Then $\mathrm{Bun}^{ss}_{r,d}(C)$ is a smooth, univ closed and irred. alg. stack of dim $r^2(g-1)$ which admits a proj. good moduli space.

- $\mathcal{M}_G(C)$ mod. space of G-bundles over a curve for a red. alg. group G. (Luca Casonia ?)

- Higgs mod space of Higgs bundles over smooth curve (Capasso)

- $\mathcal{M}_{g,n}$ stack of all genus g curves with n markings is an alg. stack

A and $\overline{A}_g^*$ toroidal compactifications of A_g

- $\underbrace{\mathcal{A}_g}$ and $\underbrace{\bar{\mathcal{A}}_g^*}$ → toroidal compactifications of $\mathcal{A}_g$
  ↳ p.p. ab. van. of dim. g

- Univ. Jacobians
- Stacks over other geometric contexts
  - → diff. stacks
  - → top. stacks
  - → cone stacks
  - → Artin fans

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- <u>Ch. 0</u>
  ↳ Int. & Motivation → funct. point of view
  → cat. theory (Yoneda Lemma)

- <u>Hilbert and Quot schemes</u> ~ Simon

- <u>Sites, sheaves and stacks</u>
  - Groth. top. and sites
  - étale top.
  - sheaves and sheafification

  } Ismaele

- Groupoids and pré-stacks
- stacks

  +

  exemples

} D. Gori

- **Alg. spaces and alg. stacks**

  . Def. alg. space, DM stacks and alg. stacks

  . $\mathcal{M}_g$ is algebraic and $\mathrm{Bun}^{n}_{r,d}$ ...

Properties of stacks
- groupoids and eq. relations
- representability of the diagonal
- dimension, tangent spaces

- Charact. of DM stacks

smoothness and formal criterion

properness and valuative criterion

$\leadsto$ $\mathcal{M}_g$ is smooth of dim $3g-3$ over $\mathrm{Spec}\,\mathbb{Z}$

$\leadsto$ $\overline{\mathcal{M}}_g$ is proper

C... of DM stacks

- Geometry of DM stacks

- Existence of coarse moduli spaces
  Keel-Mori criterion

- Good moduli spaces (Alper)
  ↳ Alper, Halpern-Leistner, Heinloth

- $\overline{M}_{g,n}$ geom. of $\overline{M}_{g,n}$

  nodal curves, stable curves
  stable reduction

  $\overline{M}_{g,n+1} \longrightarrow \overline{M}_{g,n}$ univ. family

- M_g is irreducible over $\operatorname{Spec}\mathbb{Z}$ (DM)

- $\overline{M}_g$ admits proj. coarse mod. space.

  ↳ Keel-Mori $\Rightarrow$ $\overline{M}_g$ admits a coarse mod. space in the cat. of alg. spaces

  Use Kollár ampleness lemma

to show that this mod. space
is projective.

⇝ further material depending on
the interests of participants.

Schedule

- Do not overlap w. grp. on Model
  " " Bridgeland stability
  21^{th} Feb. → 1^{st} March
- Marc de Cataldo 8^{th} March (2 weeks
  Tu, Th. 11 - 13)

⇝ Th. at 16:00
Starting March 3^{rd}